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MEMOIR

ON

A NEW AND CERTAIN METHOD

OF ASCERTAINING

THE FIGURE OF THE EARTH

BY MEANS OF

OCCULTATIONS OF THE FIXED STARS.

By A. CAGNOLI.

WITH

NOTES AND AN APPENDIX

By FRANCIS BAILY.

LONDON:

1819.

PRINTED BY RICHARD AND ARTHUR TAYLOR, SHOE LANE.

ADVERTISEMENT.

THE following Memoir originally appeared in the Transactions of the Italian Society (*Memorie di Matematica e di Fisica della Società Italiana. Tom.* vi. *Verona* 1792): but, although it has been so many years before the public, I do not find that the subject of it has been taken up by any persons either here, or on the continent. In fact, I believe that it is not generally known in *this* country: and it is with a view of drawing the attention of astronomers more closely to it, that I now present them with this translation. The acknowledged talents and abilities of the Author must at all times ensure it an attentive perusal: but particularly at this period, when a more than ordinary degree of interest is excited towards the subject of the *true figure of the Earth*. I am aware that there are some apparent difficulties in the method pointed out in the Memoir, yet I am

not without hope and expectation that they may be eventually overcome: and that the mode herein proposed may at least be brought *in aid* of the other methods at present adopted for determining that difficult problem.

To the Memoir itself I have subjoined some Notes connected with the subject; which, however, may be distinguished from those of the Author by the addition of the letter B. And to the whole I have added an Appendix, wherein I have attempted to illustrate the views of the Author; and ventured to propose such methods as may best tend to carry his object into execution.

FRANCIS BAILY.

GRAY'S INN,
May 20, 1819.

*** As the present Memoir is not published for *sale*, I shall be happy to furnish such persons, as may send their Cards for that purpose, with any number of copies they may require.

F. B.

MEMOIR

ON

A new and certain Method of ascertaining the Figure of the Earth.

By A. CAGNOLI.

§ 1. The truth of the Newtonian theory of universal gravitation is proved by the wonderful agreement in all the celestial phænomena, which have hitherto been submitted to the tests of calculation and of observation. There probably is not any astronomer of the present day who is not convinced that our planet is compressed, or flattened, at the poles, and protuberant at the equator. In fact, if we combine this theory of Newton with the hypothesis of the elliptical form of the earth, we shall find that the precession of the equinoxes and the nutation of the earth's axis are sufficiently accounted for. Other arguments indeed, in favour of the elliptical form, may be derived from the rotation of the earth, which is now generally admitted:

and others again from the differences in the length of the pendulum vibrating seconds in different latitudes. Still however we want some theory of the internal density of the earth; which, it is feared, will ever remain hid from human investigation. But if, to the want of this essential knowledge, we add the irregularity of the figure of the earth, as shown by the measurements of the degrees on its surface; if, to the doubts which some persons may still entertain of the truth of the compression of the poles*, we add the discordant opinions as to the quantity of that compression; we may surely assert that it will be no small advantage to point out a method (independent of all hypothesis) for determining, with facility and with the greatest precision, the differences between the terrestrial radii at an indefinite number of points on the earth's surface.——Such indeed is the object of the present memoir.

§ 2. It has always been said, when speaking of the compression or flattening of the poles of the earth, that the *Parallax of the Moon* would afford the best means of ascertaining it; provided the variations, arising therefrom, were of sufficient magnitude to be observed with perfect accuracy. But, since, by supposing this compression not to exceed $\frac{1}{300}$ of the semidiameter (which is the most received opinion at the present day) there would be

* See Lorgna "*Principj di Geografia &c.*" § 31.

a difference of not more than 9″ between the moon's parallax at the equator, and her parallax at a high latitude, for instance 60°; so indeed it is but too true that a difference so small might be easily concealed under the possible errors of observation. It was, on this account, that Manfredi and Maupertuis in vain suggested the determination of the compression by direct observations of the moon's parallax: nor has any advantage been hitherto obtained by this method.

§ 3. But this is not the only instance in which a quantity, extremely minute on one side, has been found to produce effects that are quite discernible on the other. The attention of the observer should be directed to those points where they are most sensible. This has been the object of my research; and I hope not entirely without success. For, there are circumstances in which scarcely a *second* in the parallax may cause a difference of 15, 20, or 30 seconds of time, or even more, in the duration of the occultation of a star by the moon. But, occultations, more particularly when the immersions take place behind the dark limb of the moon, can easily be observed without committing an error of a single second of time*: so that opportunities

* The *duration* of an occultation cannot probably be so well observed when the immersions take place behind the dark limb of the moon, as when they take place behind the illuminated side; since the *instant of emersion* cannot be so well ascertained. B.

exist not only of removing all doubt as to the reality of the compression of the poles, but also of such frequent occurrence that the gradation of the compression, (or variation of the curvature) at different latitudes, may be accurately distinguished. The importance of the subject induces me to treat it more fully in detail.

§ 4. I shall not take up the time of the reader in demonstrating that no hypothesis is required to deduce the variation of curvature from the observation of the parallax. For, since the variation of curvature is nothing more than the inequality of the terrestrial radii (and that is nothing else than the difference of the parallax), it is evident that finding by observation the variation of parallax at different latitudes is in fact finding the variation of curvature by immediate observation. One involves the other without any intermediate help. The only difficulty consists in freeing from uncertainty those observations which show the unequal parallax: and this is precisely the special character of the particular kind of occultations which I have in view. It is true that astronomers, in calculating occultations, have made the parallax vary in conformity with that quantity of compression which they have adopted as the true theory. Moreover, these variations do not in general produce differences which are discernible in the relative duration of the occultations

observed in different latitudes. Those differences, which *are* discernible, arise only in certain circumstances, to which hitherto no particular attention has been paid. It is on this account that occultations have not, as yet, served at all to determine the amount or quantity of the compression of the polar axis.

§ 5. Before I proceed to describe what these favourable circumstances are, it will be proper, in order to appreciate their utility, to define the precise degree of accuracy of which the *computation* and *observation* of occultations are capable. With respect to the *computation*, it is evident that the apparent distance between the star and the moon cannot be accurately determined unless we know exactly not only the apparent place of the star, but also that of the moon and its diameter. These elements are obtained by calculating the phænomena from observations made in a place (the longitude of which is known) under circumstances where the alteration of the parallax, on account of the variation in the curvature of the earth, does not produce a perceptible difference in the duration of the occultation: conditions not uncommon, nor difficult to be obtained.

§ 6. Let us then suppose that the moon's diameter and the place of the star are well determined (in which elements the least uncertainty exists); and

that any small errors, which may occur, be thrown on the place of the moon. Let us, moreover, endeavour to discover (by comparing the calculation with the observation) the errors of the tables in the longitude and latitude of the moon, mixed (as we have already stated) with the preceding errors*. In the present state of astronomy I do not see any great danger to be apprehended from this union of the two errors, notwithstanding the delicacy of the investigation in question. At any rate we may arrive at a sufficient degree of accuracy by verifying the place of the star by a sufficient number of observations, and also the diameter of the moon by means of the telescope which is used for the occultations.

§ 7. Having corrected the errors of the lunar tables, we shall have accurate elements wherewith to represent the state of the heavens: and there can then be no doubt that we may arrive at great exactness in our calculation of the duration of an occultation (or of the moment of immersion and emersion) for the place in which we wish to ascertain the curvature of the earth: since the geographical longitude of the place has been determined by

* See the Memoir which obtained the prize from the Academy of Copenhagen: "*Méthode pour calculer les longitudes géographiques,*" *Vérone; chez Ramanzini.*

means of occultations under circumstances in which (and they are the greater part) the variation in the curvature is not perceptible.

§ 8. With respect to the accuracy of *observations* of this nature, I have already said that the *immersions* behind the dark limb are not subject to the least error. Those on the illuminated side may be liable to some slight uncertainty if the star be not of the first or second magnitude, or if the power of the telescope be too small. The *emersions* from the dark limb are in general to be depended upon: whilst those from the illuminated side are the most doubtful of all*. But, even supposing that the error, in the last-mentioned case, may amount to 8 or even 10 seconds of time, will it be sufficient to conceal the variation of curvature altogether in the circumstances I have contemplated, and where the difference that it will produce may be ten or twenty times greater than the presumed

* It appears that, in some occultations, the immersions and emersions take place wholly on the dark side of the moon. See an account of the occultation of β *Virginis* observed by Mr. Troughton, on May 22, 1801. *Connaissance des Tems, Année* xiii. *page* 342. Sometimes an occultation may be observed when the moon itself is not visible: as in the occultation of Venus on May 13, 1801, between 8 and 9 o'clock in the morning; the moon being then only a few hours old. *Ibid. page* 417. A favourable occultation may also occur during a total eclipse of the moon, when the immersions and emersions of stars of the 6th or 7th magnitude may be distinctly observed. *Ibid. Année* ix. *page* 335. B.

error? Can we, indeed, expect to obtain more certain observations*? since a single observation of this kind accurately made in the place, for which the variation of curvature is sought, is sufficient to discover it by means of the comparison of the *calculated* with the *observed* moment.

§ 9. I shall now point out what are, in fact, the circumstances in which the differences of the parallax between two latitudes may be shown in an undoubted manner: and, in order virtually to embrace all the cases, I shall take mean quantities in the elements which enter into this investigation. Let us therefore suppose, first, that the moon's apparent semidiameter is 15′. 45″, its equatorial parallax 57′. 40″, and its horary motion 32′. 56″,5: secondly, that the occultation is observed in N. Latitude 60°, in which parallel there are three celebrated observatories, viz. *Petersburg*, *Stockholm* and *Upsal:* lastly, that the apparent height of the moon is 10°. If there be no variation in the curvature of the earth's surface the parallax of height will be 56′. 47″, 4†: but, if the polar compression amount to $\frac{1}{300}$ of the earth's radius, the same

* I find it difficult here to give a faithful translation of the author's words: the original runs thus, "Senza che ci possiamo attenere alle "osservazioni più sicure: ed una sola fase, &c. &c." B.

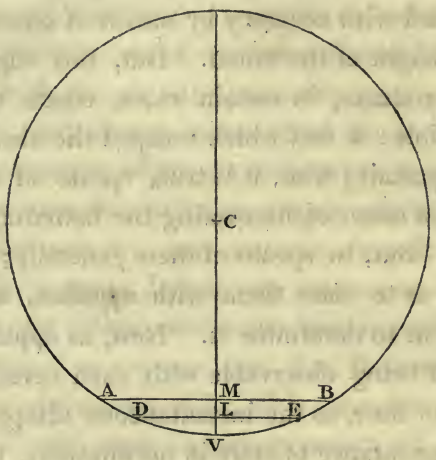
† Let p = the horizontal parallax of the moon; and h = the height of the moon: then $\pi = p.\ cos\ h$ = the parallax of height. B.

parallax will be 56′. 38″, 9 *. The parallax would therefore under these circumstances experience an alteration of 8″,5 ; a quantity certainly too small to be verified with accuracy by means of observations of the height of the moon. But, this slight alteration produces, in certain cases, effects that are very visible: a fact which escaped the observation of Maupertuis ; who, it is true, speaks of occultations as a mean of discovering the flattening of the earth†. But, he speaks of them generally ; and so slightly as to class them with appulses, as being *equally* fit to determine it. Now, as appulses are far from being observable with such certainty, in regard to time, as the instantaneous disappearance and reappearance of stars in occultations, it is evident that Maupertuis can never have had in view the particular cases which I am about to point out, and which differ materially from ordinary ones.

* Let a = the polar compression of the earth, supposed = $\frac{1}{300}$; λ = the latitude of the place ; and the radius of the equator equal to unity : then we shall have the length of any other terrestrial radius = $(1-a.\,sin^2\lambda)$ nearly; which, being multiplied by π, will give $\varpi = \pi(1-a.\,sin^2\lambda)$ = the parallax of height on the supposition that the earth is an oblate spheroid. Whence we also have $\frac{\varpi}{\pi}$ = the length of the terestrial radius at any given latitude. And if δ denote any given increase of the parallax, we shall have $\frac{\delta}{\pi}$ for the corresponding increase in the length of the Earth's radius. B.

† *Préface au discours sur la parallaxe de la lune.*

§ 10. Let us suppose AVB a portion of the circumference of the moon's disc opposite to the ob-



server; C the moon's centre; and that the radius CV, which divides the arc AVB into two equal parts, coincides with the vertical circle of the place of the observer. Let us further suppose that the line MV (or the ver-sine of the arc) is equal to 60″; and that, during the occultation, the chord AB of the moon is that which the star would apparently describe if the earth were spherical; or the chord DE that which it would apparently describe if the axis of the earth were compressed $\frac{1}{300}$. Then ML will be equal to 8″,5 as before mentioned: and, computing from the mean rate of the horary motion of the

moon, it will be found that the duration of the occultation behind AB will be 20′. 7″ of time; and, behind DE, 18′. 41″*. We therefore see how considerable is this difference of 1′.26″ of time; and how well such observations are adapted not only to convince those who doubt the reality of the compression of the earth's axis, but likewise to show with very great approximation to truth the relative length of the earth's radii in different latitudes. For, in the case just mentioned, the variation of a twenty-thousandth part of that length † will cause a difference of one second of time in the duration of the occultation.

§ 11. Moreover, it is evident that the effect, of which I have been speaking, may be much greater.

* For, joining C,A, and C,B, we shall have CV=CA=CB=15′.45″, and CM=(15′.45″−60″)=14′.45″; consequently AM=MB=$\sqrt{(AC-CM).(AC+CM)}$=5′.31″,4; which, being multiplied by $\frac{60'}{32'.56'',5}$, in order to reduce this *distance* into *time*, will give 10′.3″,6 for the time of the star's passing from A to M: therefore the time of the star's passing from A to B will be equal to 2×(10′.3″,6)=20′.7″,2. But, if the depression of the earth's axis cause a variation in the apparent height of the moon equal to 8″,5, then will CL=14′.53″,5, consequently (as CD=AC) DL=LE=$\sqrt{(AC-CL).(AC+CL)}$=5′.7″,7; which, being also multiplied by $\frac{60'}{32'.56'',5}$, will give 9′.20″,5 for the time of the star's passing from D to L: therefore the time of the star's passing from D to E will be equal to 2×(9′.20″,5)=18′.41″. B.

† Or, about 1000 feet. The mean radius of the earth being 20,898,240 English feet. B.

It increases greatly as the line MV (or the ver-sine of the arc) becomes smaller: if, for example, it were 30″, the duration of the occultation under AB would be 14′.20″; and, under DE, 12′.10″, being a difference, in the duration, of 2′.10″ of time *. In such a case, a difference of 500 feet in the length of the earth's radius might be rendered evident: since such a variation would produce a difference of one second of time in the duration of the occultation †.

§ 12. I am well aware that it scarcely ever happens that the apparent path of the star is rectilinear,

* In this case, CM = (15′.45″−30″) = 15′.15″; consequently AM = 3′.56″,2: which being multiplied by $\frac{60'}{32'.56'',5} \times 2$, as in the preceding cases, will give 14′.20″,4 for the time of the star's passing from A to B. But, supposing the depression the same as in the former instance, we should have CL = 15′.23″,5; consequently DL = LE = 3′.20″,4: which, multiplied also by $\frac{60'}{32'.56'',5} \times 2$, will give 12′.10″,2 for the time of the star's passing from D to E, as mentioned in the text. B.

† According to my calculation, a difference of only 400 feet in the length of the earth's radius would produce a difference of one second of time in the duration of the occultation. But, if the line MV were only 15″, a difference of less than 200 feet would produce a difference of one second of time in the duration. And, agreeably to these principles, it will be evident that the duration of such occultations will vary according to the elevation of the spot (from which they are observed) above the level of the sea; and will sensibly differ when observed in valleys, or on the tops of high hills and mountains. A fact which perhaps has not been sufficiently attended to, either in observations of occultations, or in calculations wherein the parallax of the moon is involved. B.

and at the same time perpendicular to the vertical circle which passes through it, at the moment of the middle of the occultation, as in the case I have supposed. But such a supposition may be readily conceded, as being a mean between the possible cases. For, if the actual phænomena do in some cases deviate therefrom, so as to render the effect of the compression *less* upon the duration of the occultation than when calculated on the above hypothesis, it will on the other hand often occur that they are *greater**. Hence I have no hesitation in asserting that the means, which I have pointed out, are not dependent on any hypothesis, and are the most certain of any that have yet been suggested for determining the variation in the curvature of the earth's surface.

§ 13. With respect to the *pendulum*, the differences in its length are not only too minute and too difficult to be measured with precision, but are also too much influenced by the unknown internal density of the earth to be brought at all into comparison with the great differences, above mentioned, in the duration of occultations.

§ 14. The different *measurements of the degrees*

* I confess I do not see the accuracy of this conclusion: for it appears to me that the *effect* of the compression, on the duration of the occultation, must be *greatest* when the apparent path of the star is perpendicular to the vertical circle. B.

of the meridian are likewise so discordant amongst themselves, that the respectable author (whom I am about to quote) asserts that the greater the number of degrees that are measured, the more uncertain does the figure of the earth seem to be*. Let us

* This indeed appears to be the case from some of the late measurements. With respect to the meridian of France, M. Delambre states (*Traité d'Astronomie, Vol.* 3. *page* 567) that "it was hoped "that the measurement of that arc (divided nearly into two equal "parts by the parallel of Evaux) would have given the quantity of "the compression of the earth's axis. The calculation, however, "makes it = $\frac{1}{130}$; a quantity much too great. The whole arc, com- "pared with that of Bouguer at Peru, makes it = $\frac{1}{334}$; which is "too small." And he goes on to observe that, from a revision of the calculations of Lacondamine, and taking a mean between him and Bouguer, he has deduced the quantity $\frac{1}{309}$; which he considers as the most probable one, and which he proposes to be adopted in all future calculations. Nevertheless he afterwards states (*Ibid. page* 572) that a "compression of $\frac{1}{180}$ represents very well the arcs be- "tween Greenwich and Paris, between Dunkirk and Paris, between "Greenwich and Evaux, between Greenwich and Barcelona, and "between Carcassonne and Mount-Jouy. It appears then (he adds) "that, without much variation, the arc of the meridian from Green- "wich to Barcelona, indicates (in the whole and in its several parts) "a compression which differs very little from $\frac{1}{180}$." And the conclu- sion, which he draws from the whole, is that "the arc of France "indicates a compression more considerable than that of the globe "in general." It is worthy of observation that Maupertuis, who was one of the astronomers that measured the arc of the meridian in Sweden in 1736, makes the compression = $\frac{1}{178}$, by comparing it with the arc then measured in France. But, M. Svanberg who, in 1805, remeasured the same arc, deduces the compression = $\frac{1}{307.4}$, when compared with the arc recently measured in France; and = $\frac{1}{312.3}$ when compared with that of Bouguer at Peru. This is

moreover bear in mind the enormous expence and labour required in measuring a degree of the earth's surface. Let us recollect the great uncertainty that may attend such a measurement from the deviation of the plumb-line made use of in the instruments employed for the astronomical observations, occasioned by mountains and valleys, subterraneous cavities and other irregularities in the upper strata

however on the presumption that the standard measure, used in the survey, was equal to the double metre at zero of the thermometer: but, if it was so only at the temperature of $+16^{\circ}\frac{4}{4}$, the corresponding compressions would be $\frac{1}{326}\cdot 9$, and $\frac{1}{331}\cdot 7$. Other results are obtained according to the different tables of refraction made use of in the calculations; so that, after all, considerable doubts exist as to the exact quantity of the compression, as deduced from that survey. M. Delambre, from the measurements, given by Major Lambton, of an arc in the East Indies, stated the compression $= \frac{1}{305}\cdot 7$: but, from the subsequent operations of the same gentleman, he afterwards deduces it $= \frac{1}{312}\cdot 8$. Major Lambton himself, however, from a more recent measurement has stated the compression to be $= \frac{1}{309}\cdot 6$, but, that on comparing the whole of his arc with the whole of the French arc, the compression would be $= \frac{1}{355}$. A remarkable discordance in the results. The measurement of Lacaille, at the Cape of Good Hope, compared with that at the equator, gives $\frac{1}{180}$. Lastly, I shall add that the recent measurement of the arc, in England, seems to indicate a *prolongation*, instead of a *compression* of the polar axis. These discordant conclusions leave a very unsatisfactory impression on the mind, notwithstanding the acknowledged talents of the persons employed in those surveys; and prevent us from placing too great reliance on a method which produces results, differing so widely from each other. The learned Boscovich might well exclaim that "the greater the number of degrees that are measured, the "more uncertain does the figure of the earth seem to be." B.

of the earth: and, of what great importance such anomalies may be, we may learn in the celebrated work of Boscovich *.

§ 15. It may perhaps be objected that the two conditions necessary for my purpose may not frequently occur together; namely, that the altitude of the moon †, and the chord of the lunar disc which passes over the star, must be small. This is probably the reason why this method of ascertaining the compression of the earth's axis has not hitherto been practised. It is true that the greater the altitude of the moon above the horizon, and the nearer to its centre the apparent path of the star, so much the smaller will be the difference of parallax caused by a sphere and an oblate spheroid ‡: and consequently the more easily might the errors of the elements

* *De Expeditione Litteraria, Lib.* v. § 230, *et seq.*

† It does not appear that the altitude of the moon should be limited to the quantity mentioned by the author in the text, since the variations, alluded to, will be perceptible at the height of 20, and even 30 degrees. Thus, in the case mentioned in § 9, it appears that the variation in the parallax of the moon at the height of 10° is 8″,5: but, if the moon were at the height of 20° above the horizon, the variation would be 8″,1; and at 30° it would be 7″,4. B.

‡ The smaller the chord of the lunar disc (or the smaller the versine of the arc) the more perceptible will be the difference in the duration of the occultation in such cases; as will appear from the following Table; which shows the duration of an occultation of the star on the supposition that the place of the star, when it passed the vertical circle, would be 60″, 30″, 15″, 10″ and 8″,5 within the moon's

made use of in the calculation be confounded with that difference. But these are precisely the occasions most favourable for the observations with which these particular occultations may be compared; and by which (as I have said) the errors of the tables are corrected, and the geographical longitude determined.

§ 16. It is true that the conditions here required, for ascertaining the flattening of the poles, cannot be very frequently obtained: but, if we look out for them with the diligence that the importance of the

disc, provided the earth were a perfect sphere: together with the corresponding durations provided the earth were an oblate spheroid, having the axis compressed $= \frac{1}{300}$. Latitude of the place 60°: height of the moon 10°.

Earth = Sphere.		Earth $= \frac{1}{300}$		Difference of Duration.
✻ within ☽'s disc.	Duration.	✻ within ☽'s disc.	Duration.	
60″	20′ 7″	51″,5	18′.41″	1′.26″
30	14.20	21,5	12.10	2.10
15	10.11	6,5	6.43	3.28
10	8.19	1,5	3.14	5. 5
8,5	7.41	0	Appulse	7.41

In the APPENDIX I have given other tables, showing the differences that would arise from varying the latitude of the place, the height of the moon, and the quantity of the earth's compression: whereby the reader may be better able to judge of the maximum of difference which would arise under the most favourable circumstances. B.

object demands, we shall perhaps meet with them more frequently than we imagine. For, I may venture to assert that there does not pass a month without the occurrence of an occultation of some star whose position is well known: and there is no occultation that will not afford, to some part or other of the earth's surface, the conditions required*. If only once out of twenty times they should occur, in a place where there is an astronomical observatory, the question as to the compression, and also as to its quantity, would be very soon determined.

§ 17. For the solution of such questions, for which immense sacrifices have hitherto been made

* Since this was written, the positions of most of the zodiacal stars have been determined with a degree of accuracy sufficient for the purposes detailed in the memoir. The labours of *Cagnoli* himself, of *Piazzi*, *Harding*, *Zach*, and *Bessell* have contributed much to this end: so that we may *now* safely assert that scarcely a *night* passes "without the occurrence of an occultation of some star whose posi-"tion is well known." Nearly forty years ago Messier made the following remark, at the close of a numerous list of observed occultations: "We see by this collection of occultations how many new ones "I have observed, in the first quarter of the moon, on the dark "limb; which are distinguished with the greatest precision. They "are frequent, and much preferable to the observations of Jupiter's "satellites, or lunar eclipses, for determining the longitude. It were "much to be wished that the conductors of our Ephemerides should "announce, for the first part of each lunation, the immersions of "stars of even the 7th, 8th and 9th magnitude, which are as readily "observed as those of the 1st, 2nd and 3rd magnitude." *Connaissance des Tems*, *Année* viii. *page* 319. B.

(for instance, in the measurement of the degrees of the meridian) it surely is not requiring too much that the trifling expense should be incurred of enabling astronomers to travel to places more favourably situated for making observations of such occultations. This would be an undertaking worthy of a sovereign who wishes to distinguish himself as a patron of science. We should, by such means, gradually arrive at a knowledge of the relative length of the terrestrial radii, in a great number of places: and, it is most probable that we might thereby be enabled finally to deduce the true and exact figure of the earth.

§ 18. Although it appears, from the different measurements of the degrees of the meridian, that the figure of the earth is not regular, still it is possible that the irregularities do not belong so much to the figure, or the radii, as to the nature of the upper strata, the different density of which may occasion the concealed error in the perpendicularity of the instruments: an error which (as I have elsewhere shown) may be quite sufficient to reconcile all the disagreements between the measurements hitherto taken*. Consequently the varia-

* Independent of the deviations arising from the causes here alluded to, the plumb-line has been sometimes known to be attracted towards the sides of the *glass* vessel, containing the weight, as powerfully as gold-leaf towards an electrical tube. The remark ap-

tions in the parallax in different latitudes might very well proceed with as much regularity as appears to exist in the variation of gravity, and in the length of the pendulum.

§ 19. It is in the power of every principal Academy materially to assist in such a discovery, by two methods. First, in regard to times *past*, to collect together, from all quarters, the observations of occultations stated to have been made in a given interval; for instance, in the last ten years: and to employ some calculator to select and compute all those which are proper for showing the variation of curvature at different places. Secondly, with respect to the *future*, to insert in the Ephemerides

pears to have been made by M. Flaugergues. "Astronomers ought "to avoid using a *glass* vessel for the water in which the weight of "the plumb-line is suspended; for, I have observed twice, in one "year, a singular deviation in the plumb-line occasioned by the at- "traction of the ball of the plumb-line towards the side of the vessel "in which it was suspended. This ball was drawn towards the side "with as much rapidity as gold-leaf is attracted by an electrified tube: "and I was obliged (in order to destroy the effect of this spontane- "ous electricity, so as to enable me to take equal altitudes) to put "a coating of sealing-wax upon the ball. But, since I have substi- "tuted a *metal* vessel, this singular phænomenon has not again oc- "curred." *Connaissance des Tems, Année* xiii. *page* 413. Although such a powerful impulse as this may not often have occurred, yet it is possible that slight deviations from the perpendicular may frequently have arisen from the cause here alluded to; and which may account for some anomalies which have been remarked in the observations made in the course of the surveys. B.

accurate notices of those places or districts where it would be most important that any occultation should be observed (particularly of the principal stars), in order that it might serve to apprize, and excite the attention of such astronomers as might be favourably situated themselves, or contiguous to more advantageous situations. It appears to me that such notices would be more important and useful than those of the phases of solar eclipses, about which the calculators of Ephemerides are in the habit of taking so much trouble.

END OF THE MEMOIR.

APPENDIX

By F. BAILY.

Since the original publication of the preceding Memoir, I am not aware that any attempts have been made to derive any practical results from the theory of the learned author; notwithstanding the simplicity of the method which he has proposed. Perhaps no practicable mode, which has been hitherto suggested for determining the true figure of the earth, or the precise quantity of the compression of the polar axis, is entirely free from errors: but, as those errors probably arise from different sources, according to the methods adopted, it is desirable that the modes of investigating it should be varied as much as possible, in order that the existing discordances may be ultimately reconciled or removed.

M. Lalande has stated (*Bibliographie Astronomique*, page 613) that in the Ephemeris of Vienna for 1791 there is "a dissertation on the figure "of the earth by M. Triesnecker; who has de-

" duced, from sixteen occultations, the compres-
" sion $= \frac{1}{329}$." Whether the occultations, which were used by M. Triesnecker in his calculations, were the peculiar sort of occultations alluded to by M. Cagnoli, I am unable to ascertain ; as I have not been able to procure a sight of M. Triesnecker's labours on this subject: but, as that work was published two years before this Memoir, I much doubt whether his method is precisely the same as that laid down by M. Cagnoli. However this may be, the design of the present translation is not at all affected thereby; as my object is to recall the attention of the public to the subject, in order that the benefit and advantage of the method may not be wholly lost; and that such scientific persons, as may be induced to co-operate therein, may look out for those peculiar occultations which are described in the preceding pages, and note down their observations thereon accordingly.

For this purpose, nothing more is requisite than a telescope sufficiently powerful to see the star distinctly, when close to the illuminated side of the moon's disc; together with a good clock, or watch, beating seconds. And although it would be desirable, in all observations of this kind, to have the *exact mean time* of the immersion and emersion of the star, yet, as it is the *duration* only from which the consequences are to be deduced, it will

be sufficient (the latitude and longitude of the place being well ascertained) if the clock or watch can be depended upon for the few minutes that the star is hid behind the moon: since the *duration* will be the same whether the clock is set to mean time, or not*.

In observing occultations, similar to those which have been the subject of the preceding Memoir, the observer should be careful not to withdraw his eye from the telescope before he is fully satisfied that the star is completely hid by the body of the moon. For the star passes behind so small a segment of the moon's disc, that it may undergo a *partial* occultation by the projecting mountains of the moon, before it is *wholly* hid by that body; as appears from the following singular phænomenon, noticed by M. Koch at Dantzic, when observing the occultation of *Aldebaran* on March 7, 1794. "He was looking out for the immersion "of the star, near the upper crescent of the moon. "It disappeared at first: but 10″ afterwards it re-"appeared suddenly in all its brilliancy. It was soon "afterwards hid a *second time*. It re-appeared how-

* In *calculating* the exact time of the immersion or emersion of a star, in order to deduce any practical and very accurate results therefrom, I do not find that any allowance is ever directed to be made for the time that *light* takes to travel from the moon to the earth; which is about *one second and a quarter*.

" ever again: but, presently after, its immersion " took place for the *third time* at $8^h.\ 28'.\ 21''$ ap- " parent time. The observer then counted $30''$; " and, finding that the star did not make its ap- " pearance again, quitted the telescope in order to " write down the observation. He came back " immediately; but the star had already emerged " from behind the moon. It is to be regretted " that he was not present at the instant of emer- " sion, in order to complete an observation which " had never been made before. However, it was suf- " ficient to show that the star did not pass a single " *second* of a degree within the moon's disc. For, " the semidiameter of the moon being $15'.\ 52''$, " and its apparent motion $29''$ in a minute, it is " found that, if the duration of the occultation " had been *one minute*, the line joining the centre " of the moon and the middle of the chord (tra- " versed by the star) ought to be $15'.\ 51''$." *Con- naissance des Tems*, *Année* vi. page 253. The writer of that article justly adds that " this is one " of the very interesting but very rare cases that " M. Cagnoli has proposed to select for the pur- " pose of determining the true figure of the earth."

It is not necessary to inform the practical astronomer that in many occultations the star, immediately before its immersion, and immediately

after its emersion, is observed to change its colour*, and for many seconds to appear not only adhering to the circumference, but oftentimes projected *on the disc*, of the moon. Whether this optical deception is caused by an atmosphere surrounding the moon, or by the instrument used for the observation†, or by what other means, I shall not stop to discuss: but, as it is probable that its effects may be more discernible in those peculiar occultations which have been the subject of the preceding Memoir, it is desirable that the particular circumstances of the case in each occultation should be noted down as they occur; together with a de-

* *Messier*, who observed the occultation of *Aldebaran* on Sept. 25, 1755, states that just before the immersion, on the illuminated side of the moon's disc, he saw the star sensibly diminish in light, change its colour and become *white*: and that for 10″ before its actual immersion it appeared adhering to the border of the moon. *Connaissance des Tems, Année* 1810, *page* 336. In a subsequent occultation of the same star, on July 11, 1757, he states that, just before its immersion, also on the illuminated side of the moon's disc, it appeared adhering to the border of the moon for 7″, and afterwards remained 2″ *on* the moon's disc, previous to its total disappearance: and that the star was easily discernible on the face of the moon by a *reddish* colour which it had acquired on approaching thereto. *Ibid. page* 339.

† In the occultation of *Aldebaran* on July 5, 1755, *Messier*, with a 4½ feet Newtonian reflector magnifying 140 times, saw the star for near 2″ *on* the moon's disc: *Le Gentil*, with a refracting telescope of 6 or 7 feet, saw it *adhering to the border* of the moon, for 4″ or 5″: whilst *Cassini*, with his 18 feet telescope, saw it *detached* from the border, to the very moment of immersion. *Ibid. page* 335.

scription of the telescope used for the observation. Astronomers indeed now adopt the method of noting down the moment of apparent *contact*, as well as the moment of the *disappearance* of the star: and when it emerges again from behind the moon, they likewise note down not only the moment of its *re-appearance*, but also the moment of its *separation* from the moon's disc. Indeed every remarkable phænomenon which may occur during *any* observation ought to be carefully registered: as it is only by a comparison of such cases that we can ultimately expect to discover the causes which influence or produce them.

M. Cagnoli, in his Memoir, seems to have addressed his remarks chiefly to the principal *observatories* of Europe. But, however desirable it may be to have the co-operation of those scientific bodies, it is evident that many private individuals may do *much* towards accomplishing the object and design of the author. In many cases, they may perhaps be able to do *more* than can be effected in any fixed observatory: since they may be more favourably situated, on the earth's surface, for the observation, or may enjoy the advantage of a more clear and favourable atmosphere. Indeed, the greater the number of observers attached to this duty, the greater will be the chance of an accurate result: for, independent of taking a mean of the observations made

near the same spot, we might be enabled to detect the errors of the lunar tables by means of simultaneous observations made in a favourable situation in a distant part of the world; agreeably to what the author has suggested in § 15 of his Memoir. In fact, by increasing the number of observers to an indefinite extent, over the whole surface of the globe, almost *every* occultation that occurs might be brought in aid of the method proposed by M. Cagnoli.

In the present contracted state, however, of the science of Astronomy, we must confine our views to a more limited scale of assistance. In this country, indeed, there are many persons scattered throughout the kingdom who have the means and opportunity of making observations of this kind: and, amongst the numerous occultations which they observe, if they should find only *one* that belongs to that peculiar class of occultations, which is alluded to in the preceding pages, it would be a valuable discovery; and ought to be noted down accordingly, with all the circumstances attending it, in order that it may be compared with others of a similar kind.

It is much to be lamented, however, that in this country there is no association of scientific persons formed for the encouragement and improvement of Astronomy. In almost all the other branches of

the Arts and Sciences, institutions have been formed for the purpose of promoting and diffusing a general knowledge of those particular subjects; such as Botany, Agriculture, Chemistry, Anatomy, Geology, &c. &c.: the beneficial effects of which are too evident to be insisted upon in this place. But, Astronomy, the most interesting and sublime of all the sciences (and, to our country, certainly the most useful) cannot claim the fostering aid of any society. It is however well known that many individuals, in various parts of the kingdom, have of late years erected and furnished private observatories at a very considerable expense. Nevertheless the utility of those establishments must be greatly circumscribed through the want of some mode of general communication amongst observers, by means of which their labours might be collected and registered; and thus rendered permanently useful. The formation of an ASTRONOMICAL SOCIETY would not only afford this advantage, but would in other respects be attended with the most beneficial consequences. It would induce many intelligent persons (who are at present restrained by want of due encouragement) to come forward as the patrons and followers of the science; and excite the more experienced to further exertions and new discoveries*.

* The name of the *Royal Society* will naturally occur to the reader on this occasion: but that society was formed for the promo-

Although much has been already done towards perfecting our present system of astronomy, much still remains to be effected, both in theory and practice. Not only would the interchange of sentiments and remarks on the various celestial phænomena, which such a Society would afford, lead to the advancement of the science in general; but the comparison and discussion of the merits of the several instruments used for astronomical purposes would also tend either to their improvement or to the discovery of new ones. By means also of a society of this kind an active communication might be kept up with the principal astronomers in different parts of the world: and, thus, continual subjects of research or observation be submitted to public attention. The funds of such a society might likewise be usefully employed in the formation of an *Astronomical Library*, consisting of the most rare and valuable books on the science; and in collecting and transcribing the observations of astronomers in various parts of the world*. But, even without any views or pre-

tion and encouragement of science *in general*; and the subject of Astronomy appears to form but a small portion of its labours. Nevertheless the *Astronomical Society* would, in common with the *Linnæan*, *Geological* and other scientific societies, hope for the *co-operation* of its learned members.

* The observations and remarks of many eminent astronomers, both ancient and modern, still exist only in manuscript, in the hands of private individuals; and ought, if possible, to be rescued from their present precarious situation, and deposited in a place of safety and

tensions to new discoveries, or to the improvement of optical instruments and time-keepers, or to the formation of such a library, an Astronomical Society might render very important benefits to Geography and Navigation by simultaneous observations of lunar distances, eclipses, occultations, and various other phænomena: which are at present in a great measure neglected because the public attention is not specially directed thereto. And the more numerous and widely diffused the members of such a society might be, the greater the probability of obviating the baneful effects of our cloudy atmosphere; an occurrence which often renders the efforts of the finest observatory totally unavailing.

convenient reference. *Bulliald* made an immense collection of observations of this kind, many of which came into the possession of *Lemonnier*; who put them into the hands of *Pingré*, for the purpose of publication. *Pingré* had himself collected also an astonishing number of rare and valuable manuscripts from all parts of Europe, and had not only calculated and reduced their contents; but had likewise discussed and compared the observations, so as to render it a work of the highest utility to astronomers. It was the labour of thirty years! and the National Assembly in 1791 issued a decree for printing it, under the title of *Annales Célestes du dix-septième siècle.* In the year 1794, about 360 pages of this work were printed; being one third only of the intended publication. The learned author died in 1796 at the advanced age of 85: since which period I do not find that the progress of the work has been at all advanced. Many valuable observations of the more recent astronomers, such as *Halley, De L'isle, Lemonnier, Godin* and others, it is well known, exist also only in manuscript; and are now mouldering in the dust, a disgrace to the nineteenth century.

There is yet another source of assistance which we may hope for in this country. It is well known that, on the continent, great advantage is rendered to astronomy by means of the *Ephemerides*, which are published under the authority of Government: and those annual publications contain a vast fund of valuable information on the subject of astronomy, which otherwise might be for ever lost to the world. The Board of Longitude in this country have *now* the power and the means* of affording

* Extract from the recent Act, 58 Geo. III. cap. 20, § 6. "And "whereas it is expedient that the said Commissioners should be en- "abled to expend certain sums towards making experiments of in- "struments, modes or proposals, and for making and publishing "observations, calculations, and tables for ascertaining the longitude, "or towards improving or correcting such as may have been already "made, or *for other purposes useful to navigation*; be it enacted that "they may pay or expend any sum or sums of money, not exceed- "ing £1000 in any one year, towards the making, correcting or "publishing any such experiments, modes, observations, calcula- "tions, or tables.

§ 7. "And whereas it is expedient that the said Commissioners "should be enabled to cause to be ascertained, as accurately as "may be, the latitude and longitude of places, whereof the exact "situation hath not been already sufficiently ascertained; be it en- "acted that they may expend or cause to be expended any sum, not "exceeding in the whole £1000, in any one year, for that purpose.

§ 8. "And whereas it may happen that proposals, inventions and "tables, or corrections and amendments of former proposals, inven- "tions or tables, ingenious in themselves and *useful to science*, and "which may *deserve encouragement*, (though they do not come within "the limits and conditions specified for the above-mentioned rewards) "may be made to the said Commissioners; and it is expedient that

similar assistance by enlarging the original plan and design of the *Nautical Almanac*, and by assimilating it to those which are published at Paris, Berlin, Vienna, and other places: a measure which would tend to retrieve the character of the work, and redound to the honour of the country*.

With the very ample funds (£4000 per annum) which Government has placed at the disposal of

" they should be enabled to bestow such moderate rewards upon the " person or persons who may have made such proposal, invention, " or correction; be it therefore enacted that the said Commissioners " may pay or cause to be paid such sum, not exceeding £500 to " any one person for any one proposal or invention, or £2000 in one " year, as they may consider the said proposals, inventions, tables or " corrections to deserve."

* Amongst the several amendments which might be made to that useful work, I would suggest the propriety and advantage of having the Right Ascension and Declination of the moon inserted to *seconds* of a degree; similar to the method *now* adopted in the *Connaissance des Tems*. This would facilitate the finding of the apparent place of the *moon*, for the purpose of comparing it with the place of any given *star*, the position of which is always given in Right Ascension and Declination: and would moreover obviate the necessity of finding the position of the *Nonagesimal*,—an unnecessary and troublesome process. As the original calculations of the computer must extend to *seconds*, there is no good reason for omitting them in the publication. I would likewise (as connected with the subject of this Memoir) take the liberty of suggesting the propriety of publishing annually in that work, an accurate list of all the Zodiacal stars (including even those of the 9th magnitude) not only in Right Ascension and Declination, but also in Longitude and Latitude, with their annual and secular variations, corrected from the latest observations. The expense would be trifling, compared with the advantage to be derived from the result.

this new Board, for scientific purposes, united to the known abilities and zeal of its several members, we may confidently hope that a new impulse will soon be given to the progress of astronomy in this country: and that, by emulating our scientific neighbours on the continent, we may avoid the lamentable necessity of resorting to them for almost all our means of deducing the situation of the heavenly bodies, and of investigating the laws which govern their motions*. Probably the object of the legislature would be more effectually answered if the new Board of Longitude would undertake to *lead* and *direct* the attention of the scientific world to such particular objects as they might think most worthy of encouragement; rather than to *wait for proposals* on every subject (useful or visionary) that may be laid before them. The *publication* of specific rewards—for the formation of new Tables of the Sun, Moon, or any of the Planets †;—for the best Essays on any particular subject of Practical or Theoretical astronomy;—for any valuable improvement in Time-pieces, Telescopes, Micrometers, or other astronomical instruments;—for the best Engravings of

* It is too well known that *all* our astronomical Tables, *all* our Catalogues of stars, and *most* of our astronomical Formulæ, in present use, are the production of the continent.

† I believe there are no tables whatever of two of the newly discovered planets.

any portion of the celestial sphere, and particularly of the zodiacal stars*;—for reducing the Observations of any celebrated astronomer;—for the Discovery of any New planet, comet, fixed star, or other remarkable celestial phænomenon;—for the Translation of any valuable astronomical treatise into the English language;—and, in general, for any other object which may be "*useful to science* and which may *deserve encouragement;*"—would necessarily excite and fix the public attention to those subjects, and perhaps more effectually promote the views of a liberal and enlightened Government.

But to return to the subject of the present Memoir.—It has been stated by M. Cagnoli, § 4, that the difference of parallax arises from the inequality of the terrestrial radii; and that the one is the measure or consequence of the other. Therefore, in order to determine the parallax at any given point of the earth's surface, it is necessary to know *the distance of that point from the centre of the earth.* But, that the reader may see the anomalies which arise on this subject, (from the actual measurement of the degrees of the meridian in different countries, and from different the-

* Accurate engravings of the Zodiacal stars, upon a large scale, would enable astronomers with greater certainty to look out for occultations of fixed stars by the moon; the observations of which are undoubtedly the best method of determining the longitude of places.

ories which have been assumed) I have calculated the following table, which shows the radius of the earth in the latitude of Greenwich (51°. 28′. 40″), according to the several hypotheses of the compression of the earth's axis therein stated.

No.	Hypothesis.	Radius in feet.	Difference.
1	$\frac{1}{139}$	20 827 239	6609
2	$\frac{1}{150}$	20 833 848	14376
3	$\frac{1}{180}$	20 848 224	7115
4	$\frac{1}{200}$	20 855 339	8350
5	$\frac{1}{230}$	20 863 689	4455
6	$\frac{1}{250}$	20 868 144	8675
7	$\frac{1}{300}$	20 876 819	1101
8	$\frac{1}{309}$	20 877 920	2640
9	$\frac{1}{330}$	20 880 560	

The first value here given arises from the recent measurement of the arc in France, by taking the middle arc at Evaux, which makes the compression of the earth's axis $=\frac{1}{139}$; and the length of the earth's radius in the latitude of Greenwich equal to 20 827 239 feet. By comparing this value with the last in the table, which is one of the hypotheses

assumed by the *Bureau des Longitudes* in the recent tables of the Moon by M. Burg, it will be found that there is a difference of no less than 53 321 feet, or more than 10 miles; a quantity sufficiently great to be detected by the observation of the moon's parallax, and which would produce a very sensible effect on the duration of such occultations as have been the subject of the preceding Memoir: since we have seen that, in some cases, a difference of less than 200 feet will produce a difference of one second in the duration.

But, although such occultations may clearly show that the earth is compressed at the poles, and although they may be effected by quantities which differ so widely from each other as in the case just mentioned; yet it may be doubted whether they are capable of determining the *precise quantity* of that compression; and therefore whether they are more eligible for that purpose than the methods at present in use. In reply to which, it may be repeated that this plan is at present proposed only *in aid* of the modes hitherto adopted for determining the true figure of the earth; and by no means as supplying the place of other methods for deciding that difficult problem.

In order, however, that the reader may have a more comprehensive view of this subject, and be enabled to see the differences which would arise in

various latitudes and under various circumstances, according to two nearly similar hypotheses of the earth's compression compared with the hypothesis of its being a perfect sphere, I have inserted the three following tables: which show the alteration in the position of the star within the moon's disc, and its consequent effect on the duration of the occultation, according to the three several hypotheses of the earth being considered as a perfect sphere, and of its being compressed at the poles $\frac{1}{300}$, and $\frac{1}{330}$; at the several heights of 10°, 20° and 30° of the moon above the horizon; and at the several latitudes of 50°, 60°, and 70°. The chord of the star's apparent path behind the moon being, in all these cases, assumed to be perpendicular to the vertical circle of the place.

Thus, by an inspection of Table II, it will be seen that in latitude 60°, when the moon is 10° high, and the star 60″ within the moon's disc, the duration of the occultation would be 20′. 7″, if the earth were a perfect sphere. But, if the polar axis were compressed $\frac{1}{300}$, the star would then be only 51″,5 within the moon's disc, and the duration of the occultation only 18′. 41″: which is the case alluded to by M. Cagnoli § 10. Or, if the polar axis were compressed only $\frac{1}{330}$, the star would then be 52″,3 within the moon's disc, and the duration of the occultation would be 18′. 50″;

being a difference of 9″ between the results of the two hypotheses. This *difference* would be the same, if the moon were 20° high; as may readily be seen by an inspection of the same table: it would also be the same under corresponding circumstances, if the observation were made in latitude 70°: as may be seen by an inspection of Table III.

This difference however may probably be considered as too small to enable us to derive any satisfactory result as to the *precise quantity* of the compression of the earth's axis. But, if we take the position of the star within the moon's disc equal to 10″ only, and the moon at the height of 10° as before, we shall have the duration of the occultation 8′. 19″ on the supposition that the earth is a perfect sphere. Then, supposing the axis to be compressed $\frac{1}{300}$, the star would be only 2″,3 within the moon's disc, and the duration of the occultation only 4′. 0″: or, supposing the axis to be compressed $\frac{1}{100}$, the star would be only 1″,5 within the moon's disc, and the duration only 3′. 14″; being a difference of 46″ in the result of the two hypotheses. If the moon were 20° high, the difference would be only 42″. If however the observation were made in latitude 70°, and the moon at 10° high, the difference would be much greater: since, on the supposition of the compres-

sion $= \frac{1}{360}$, the duration would be 2′. 30″; whereas, on the supposition of the compression $= \frac{1}{300}$ the star would not undergo any occultation at all, but would merely *touch* the moon's disc. Nearly similar results might be obtained, under particular circumstances, in lower latitudes; as may be seen from an inspection of the Tables. And it will thus be evident that a *very small* difference in the ellipticity of the earth, may, under some very favourable circumstances, be rendered sensible to observation.

It is unnecessary however to dwell farther upon this subject at present: a fair experiment does not appear to have ever yet been made of the ingenious mode proposed by the illustrious author of the Memoir, for determining the true figure of the earth. It seems to me to be highly deserving of encouragement: and my object will be fully answered if any practical good should result from the public attention which I may have excited in its favour.

F. B.

TABLE I.

Latitude 50°.

Height of the Moon.	Earth = Sphere		Earth = $\frac{1}{330}$		Earth = $\frac{1}{300}$	
	✱ within ☽'s disc.	duration.	✱ within ☽'s disc.	duration.	✱ within ☽'s disc.	duration.
	60″	20′ 7″	54″,0	19′ 7″	53″,4	19′ 1″
	30	14.20	24,0	12.51	23,4	12.41
10°.	15	10.11	9,0	7.54	8,4	7.38
	10	8.19	4,0	5.16	3,4	4.52
	6,6	6.46	0,6	2. 2	0	Appulse
	60″	20′ 7″	54″,3	19′ 10″	53″,7	19′ 4″
	30	14.20	24,3	12.56	23,7	12.46
20°.	15	10.11	9,3	8. 2	8,7	7.46
	10	8.19	4,3	5.28	3,7	5. 4
	6,3	6.37	0,6	2.2	0	Appulse
	60″	20′ 7″	54″,7	19′ 14″	54″,2	19′ 9″
	30	14.20	24,7	13. 2	24,2	12.54
30°.	15	10.11	9,7	8.12	9,2	7.59
	10	8.19	4,7	5.43	4,2	5.24
	5,8	6.21	0,5	1.52	0	Appulse

Table II.

Latitude 60°.

Height of the Moon.	Earth = Sphere.		Earth = $\frac{1}{330}$		Earth = $\frac{1}{300}$	
	✻within ☽'s disc.	duration.	✻within ☽'s disc.	duration.	✻within ☽'s disc.	duration.
	″	′ ″	″	′ ″	″	′ ″
	60	20. 7	52,3	18.50	51,5	18.41
	30	14.20	22,3	12.23	21,5	12.10
10°.	15	10.11	7,3	7. 7	6,5	6.43
	10	8.19	2,3	4. 0	1,5	3.14
	8,5	7.41	0,8	2.22	0	Appulse
	″	′ ″	″	′ ″	″	′ ″
	60	20. 7	52,7	18.54	51,9	18.45
	30	14.20	22,7	12.30	21,9	12.17
20°.	15	10.11	7,7	7.19	6,9	6.55
	10	8.19	2,7	4.20	1,9	3.38
	8,1	7.30	0,8	2.22	0	Appulse
	″	′ ″	″	′ ″	″	′ ″
	60	20. 7	53,2	18.59	52,6	18.52
	30	14.20	23,2	12.38	22,6	12.29
30°.	15	10.11	8,2	7.33	7,6	7.16
	10	8.19	3,2	4.43	2,6	4.15
	7,4	7.10	0,6	2. 2	0	Appulse

TABLE III.

Latitude 70°.

Height of the Moon.	Earth = Sphere.		Earth = $\frac{1}{330}$		Earth = $\frac{1}{300}$	
	✱within ☽'s disc.	duration.	✱within ☽'s disc.	duration.	✱within ☽'s disc.	duration.
	60″	20′ 7″	50″,9	18′ 34″	50″	18′ 2″
10°.	30	14.20	20,9	12. 0	20	11.44
	15	10.11	5,9	6.24	5	5.54
	10	8.19	0,9	2.30	0	Appulse
	60″	20′ 7″	51″,3	18′ 39″	50″,5	18′ 30″
	30	14.20	21,3	12. 7	20,5	11.53
20°.	15	10.11	6,3	6.37	5,5	6.11
	10	8.19	1,3	3. 0	0,5	1.52
	9,5	8. 7	0,8	2.22	0	Appulse
	60″	20′ 7″	52″	18′ 46″	51″,2	18′ 38″
	30	14.20	22	12.18	21,2	12. 5
30°.	15	10.11	7	6.58	6,2	6.34
	10	8.19	2	3.44	1,2	2.54
	8,8	7.49	0,8	2.22	0	Appulse

THE END.

Printed by R. and A. Taylor, Shoe-lane, London.

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